Week 10

TAM 210/211 Introduction of Statics

End of Semester Survey (TAM 210)

- The last week of discussion for TAM 210 students
- Scan QR code to access to the survey
- Complete the survey for 3 mins (only for 210 students)

WEEK 10 – Internal Forces

- **External forces vs. internal forces**
  - External forces: exerted from the outside environment

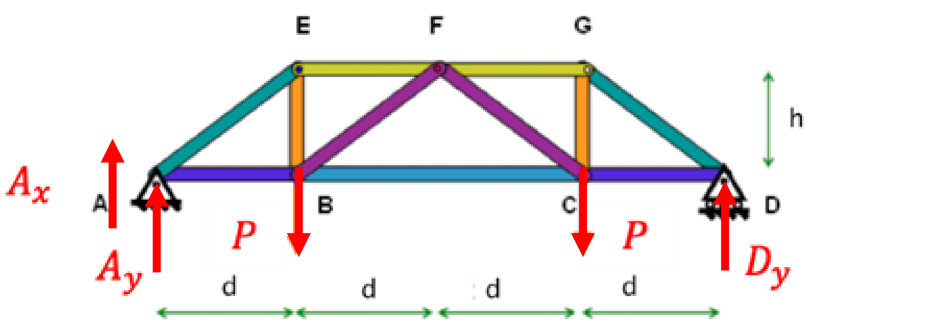
    i.e. applied loads, gravity, contact/pressure, support reaction

    ** FBD for the entire system: **only external forces** are shown
  - Internal forces: developed within members to balance with external forces

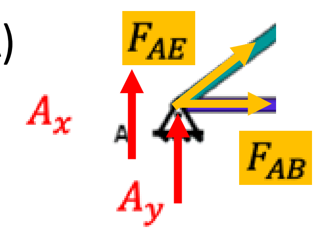
    i.e. tension, compression, axial forces, shear forces and bending moments ↑ today (Week 10)

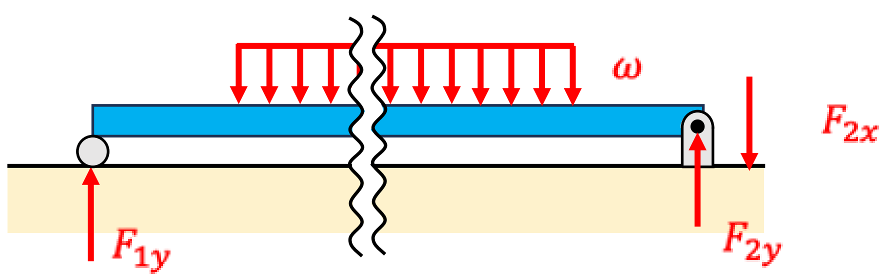
    (Week 8)

    ** FBD for an isolated component (after cut): **external forces and internal forces** appear

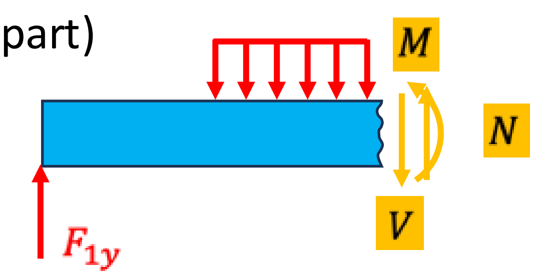


Point A)



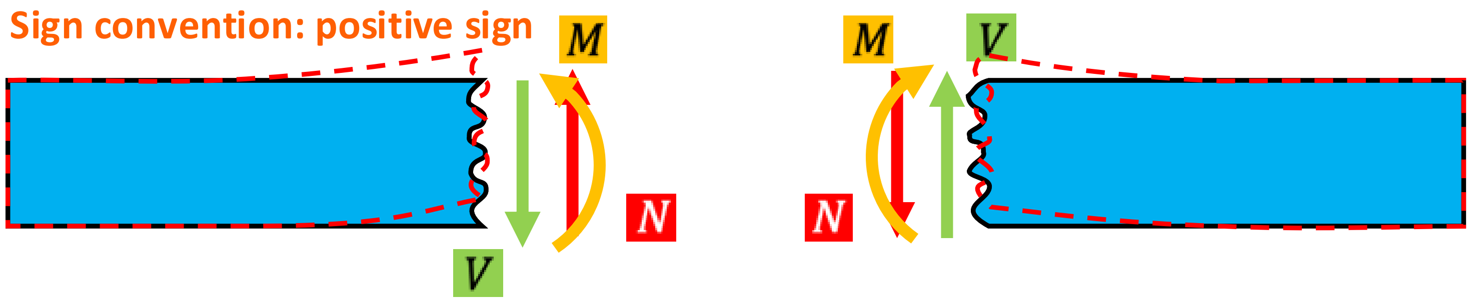


Cut part)



WEEK 10 – Internal Forces

- **Internal forces: axial forces, shear forces, and bending moments**
    - Axial forces (= normal forces) → N
    - Shear forces → V
    - Bending moment → M



- **Relations between distributed load, shear and moment**

→ more information: Engineering Statics Online Textbook

$$\frac{dV(x)}{dx} = \omega(x) \quad \longleftrightarrow \quad \Delta V = \int \omega(x)\, dx$$

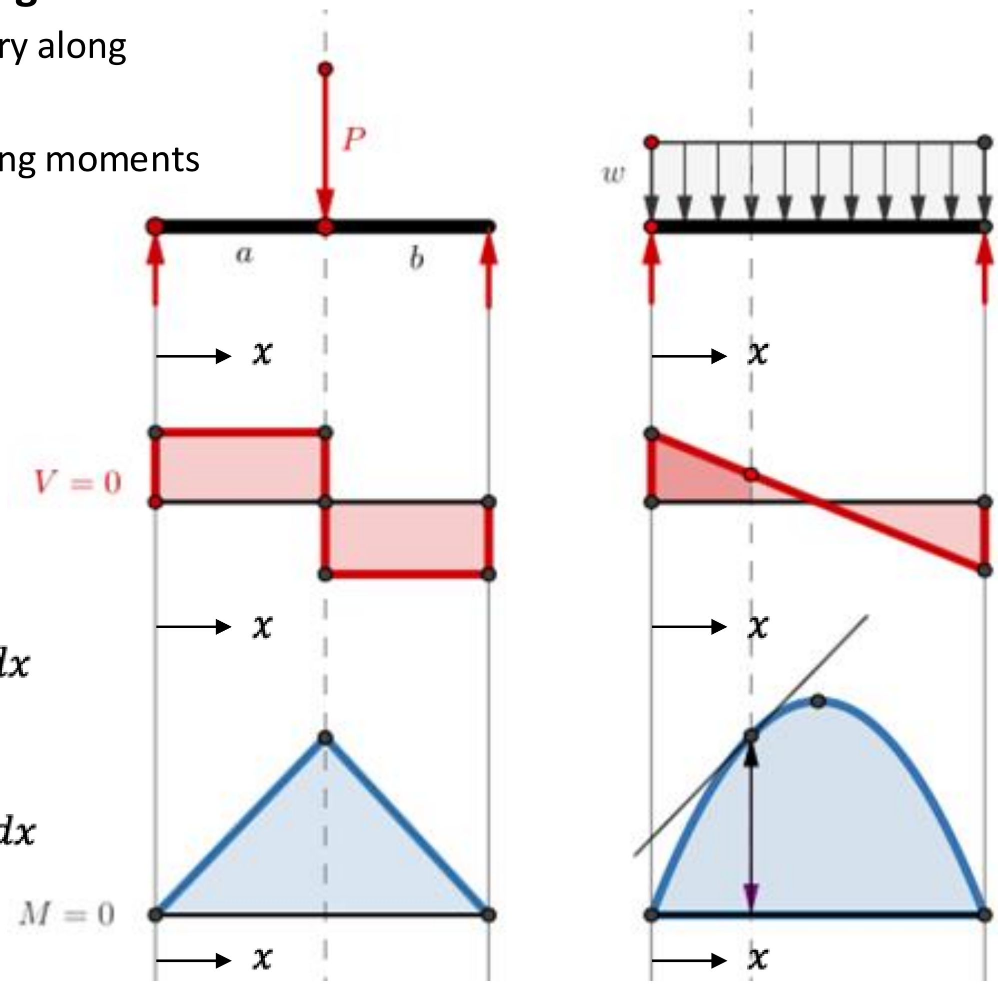
$$\frac{dM(x)}{dx} = V(x) \quad \longleftrightarrow \quad \Delta M = \int V(x)\, dx$$

WEEK 10 – Internal Forces

- **Shear force and bending moment diagrams**
  - Shear forces and bending moments vary along the length of the beam (x-direction)
  - Using diagram, shear forces and bending moments can be described graphically

$$\frac{dV(x)}{dx} = \omega(x) \longleftrightarrow \Delta V = \int \omega(x)\, dx$$

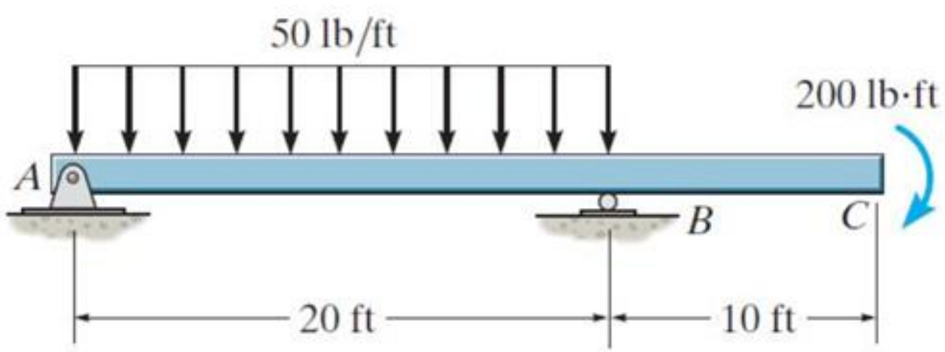
$$\frac{dM(x)}{dx} = V(x) \longleftrightarrow \Delta M = \int V(x)\, dx$$



Reference) https://engineeringstatics.org/graphical_method.html

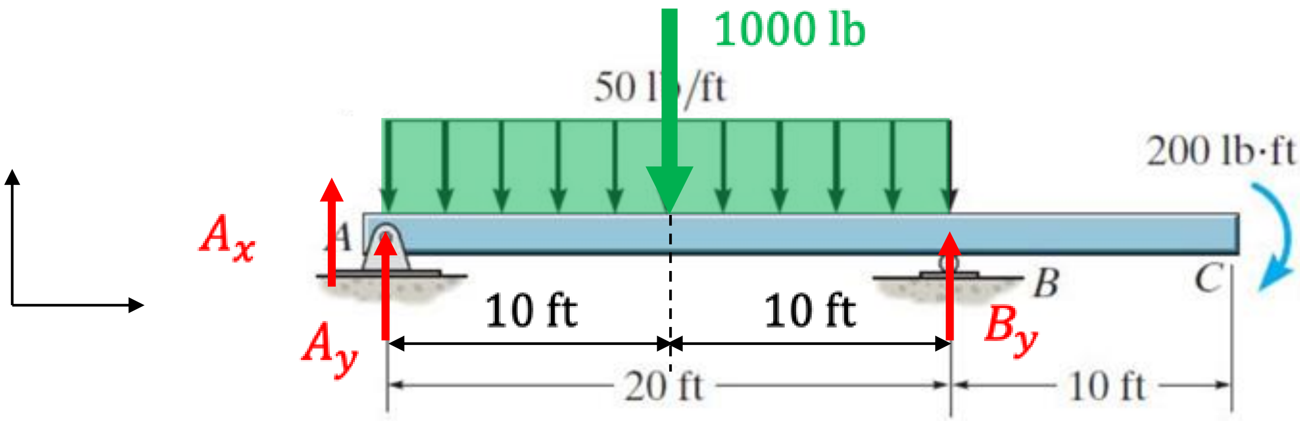
WEEK 10 – Internal Forces

- **Let's solve the first problem together!** ** Worksheet 10 p.2 problem **
  - Find support reactions
  - Specify the x-coordinates
  - Divide the beam into regions: one region for every change in loading or moment
  - Analyze each region by cutting inside each region and drawing the free body diagram of the cut segment
  - Apply equations of equilibrium to solve for V and M as functions of x



WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
  - Find support reactions
    1. Define corresponding support reactions depending on type of connections
    2. Replace distributed load to concentrated force and find its point of application
    3. Find support reactions by solving the equations of equilibrium



$$F_R = 50\ \text{lb/ft} \times 20\ \text{ft} = 1000\ \text{lb}$$

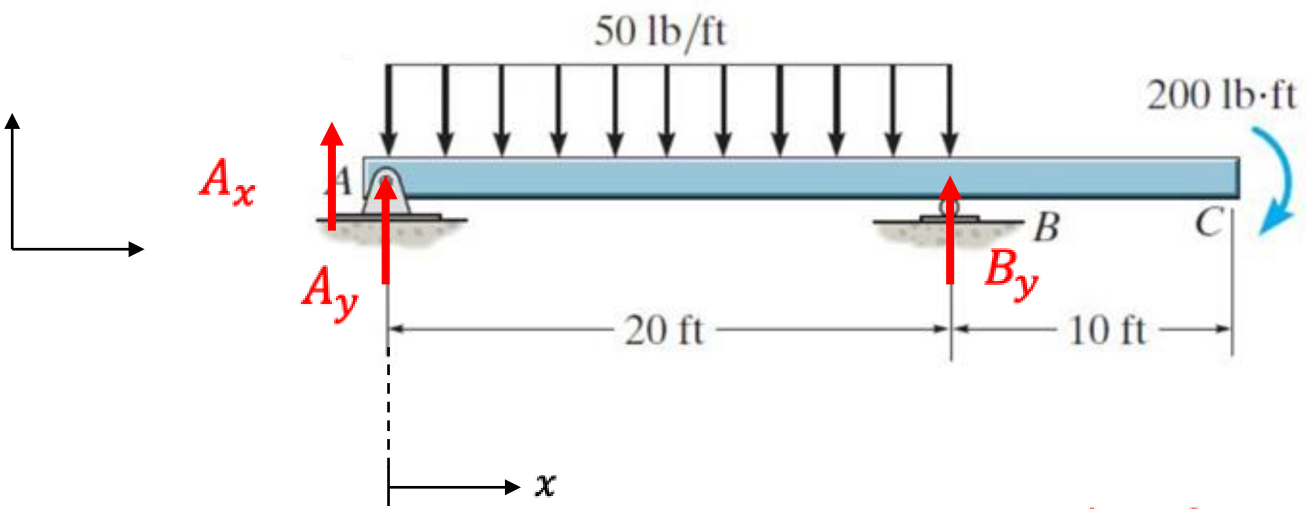
$$\sum F_x = 0: A_x = 0 \qquad \therefore A_x = 0$$

$$\sum F_y = 0: A_y - 1000\ lb + B_y = 0 \qquad \therefore A_y + B_y = 1000\ \text{lb} \qquad \therefore A_y = 490\ \text{lb}$$

$$\sum M_A = 0: -1000\ \text{lb} \times 10\ \text{ft} + B_y \times 20\ \text{ft}\ - 200\ \text{lb}\cdot\text{ft} = 0 \qquad \therefore B_y = \frac{10200\ \text{lb}\cdot\text{ft}}{20\ \text{ft}} = 510\ \text{lb}$$

WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
  - Specify the x-coordinates
    1. along the length of a beam

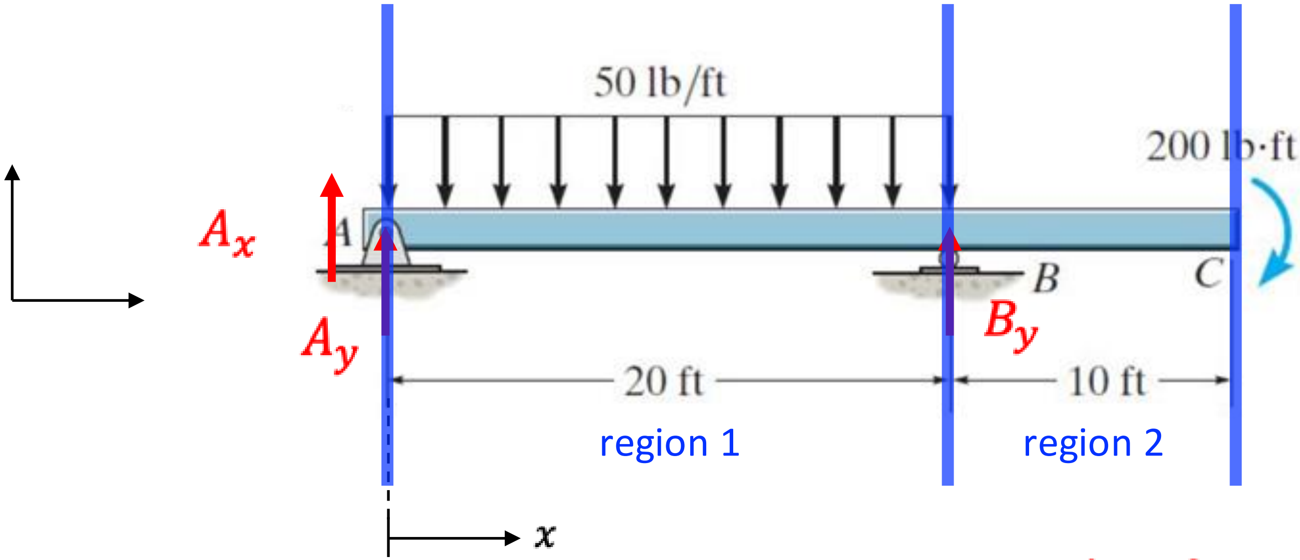


$\therefore A_x = 0 \quad \therefore A_y = 490 \text{ lb}$

$\therefore B_y = 510 \text{ lb}$

WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
  - Divide the beam into regions: one region for every change in loading or moment

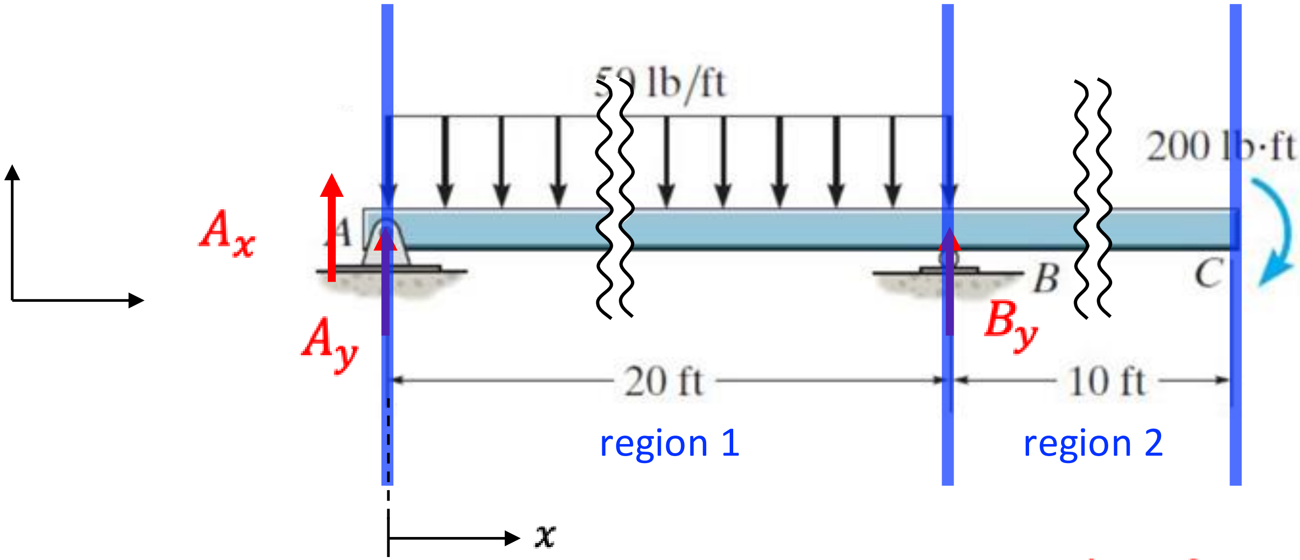


$\therefore A_x = 0 \quad \therefore A_y = 490 \text{ lb}$

$\therefore B_y = 510 \text{ lb}$

WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
  - Analyze each region by cutting inside each region and drawing the free body diagram of the cut segment



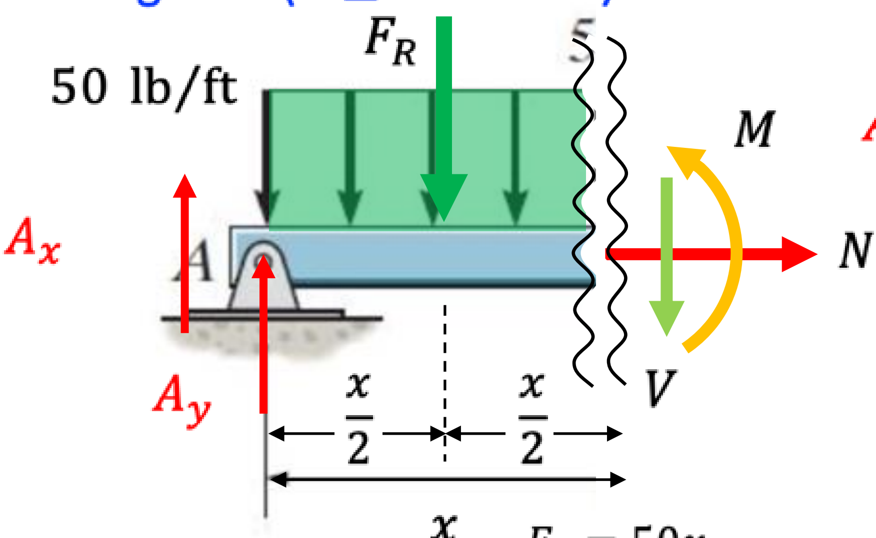
$\therefore A_x = 0$ $\quad \therefore A_y = 490$ lb

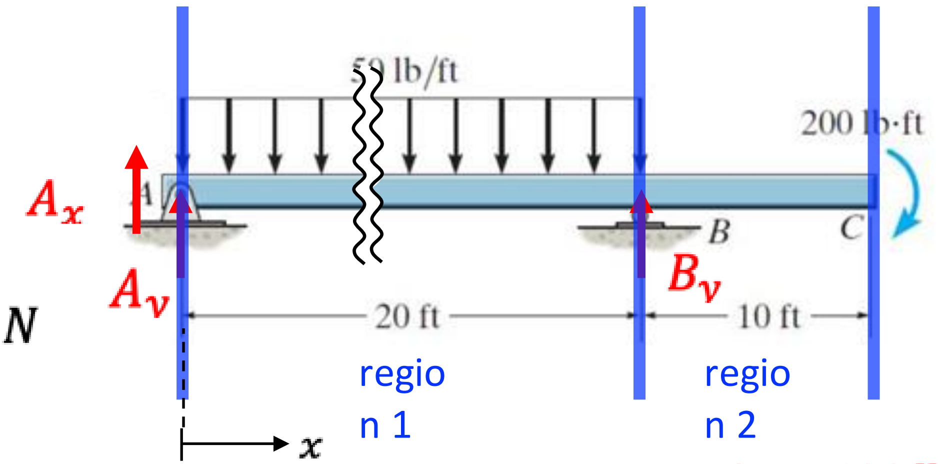
$\therefore B_y = 510$ lb

WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
  - Analyze each region by cutting inside each region and drawing the free body diagram of the cut segment
  - Apply equations of equilibrium to solve for V and M as functions of x

▪ region 1 ($0 \le x < 20$ ft)





$\therefore A_x = 0 \quad \therefore A_y = 490$ lb

$\therefore B_y = 510$ lb

$F_R = 50x$

$\sum F_x = 0: A_x + N = 0$ $\qquad \therefore N = 0$

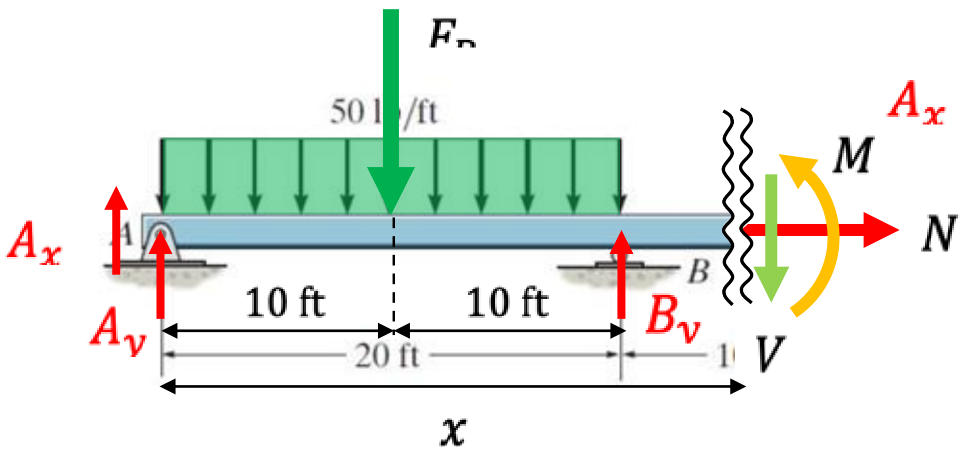
$\sum F_y = 0: A_y - 50x - V = 0$ $\qquad \therefore V(x) = -50x + 490$

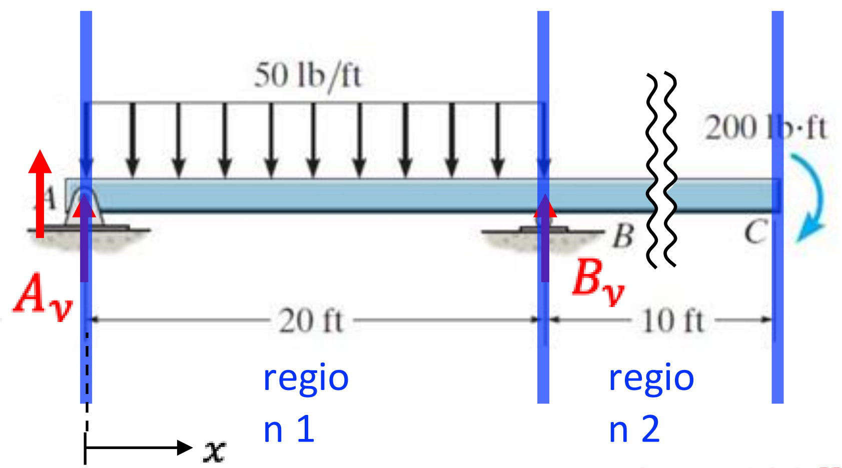
$\sum M_A = 0: -50x \cdot \frac{x}{2} - Vx + M = 0$ $\qquad \therefore M(x) = -25x^2 + 490x$

WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
  - Analyze each region by cutting inside each region and drawing the free body diagram of the cut segment
  - Apply equations of equilibrium to solve for V and M as functions of x

- region 2 ($20 \le x < 30$ ft)





$\therefore A_x = 0$ $\therefore A_y = 490$ lb

$\therefore B_y = 510$ lb

$$F_R = 1000 \text{ lb}$$

$$\sum F_x = 0: A_x + N = 0 \qquad \therefore N = 0$$

$$\sum F_y = 0: A_y - 1000 + B_y - V = 0 \qquad \therefore V(x) = 0$$

$$\sum M_A = 0: -1000 \cdot 10 + B_y \cdot 20 - Vx + M = 0 \qquad \therefore M(x) = -200 \text{ (lb} \cdot \text{ft)}$$

WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
  - Analyze each region by cutting inside each region and drawing the free body diagram of the cut segment
  - Apply equations of equilibrium to solve for V and M as functions of x

  a) Determine the internal shear force function, $V(x)$.

    - region 1 ($0 \leq x < 20$ ft)

      $\therefore V(x) = -50x + 490$

    - region 2 ($20 \leq x < 30$ ft)

      $\therefore V(x) = 0$

WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
  - Analyze each region by cutting inside each region and drawing the free body diagram of the cut segment
  - Apply equations of equilibrium to solve for V and M as functions of x

  b) Determine the internal bending moment functions, $M(x)$.

  - region 1 ($0 \leq x < 20$ ft)

    $\therefore M(x) = -25x^2 + 490x$

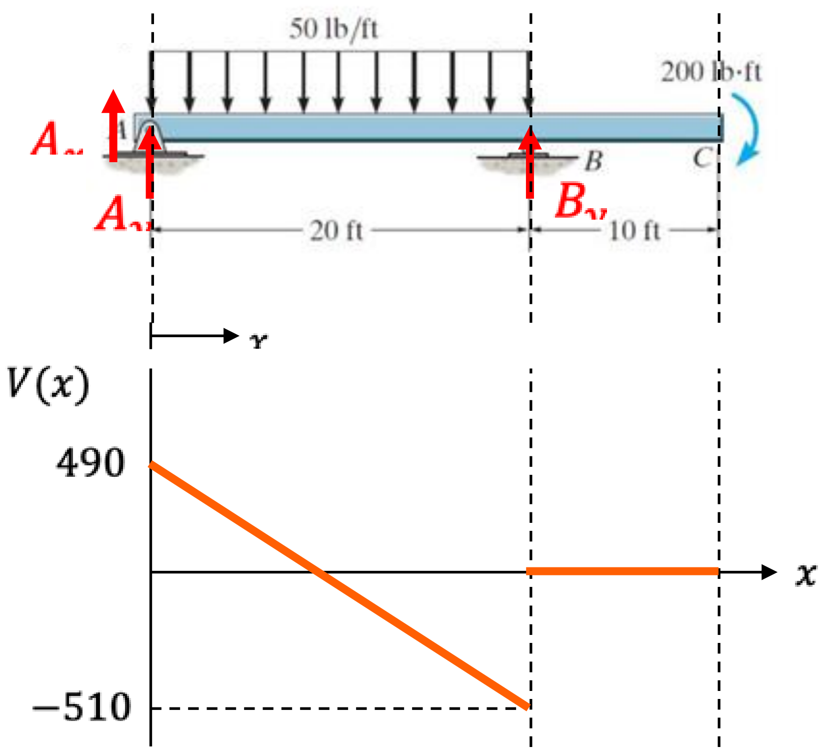
  - region 2 ($20 \leq x < 30$ ft)

    $\therefore M(x) = -200$

WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
  - Analyze each region by cutting inside each region and drawing the free body diagram of the cut segment
  - Apply equations of equilibrium to solve for V and M as functions of x

c) Use the results from parts (a) and (b) to draw the shear and bending moment diagrams.



- region 1 ($0 \le x < 20$ ft)

  $\therefore V(x) = -50x + 490$

- region 2 ($20 \le x < 30$ ft)

  $\therefore V(x) = 0$

WEEK 10 – Internal Forces

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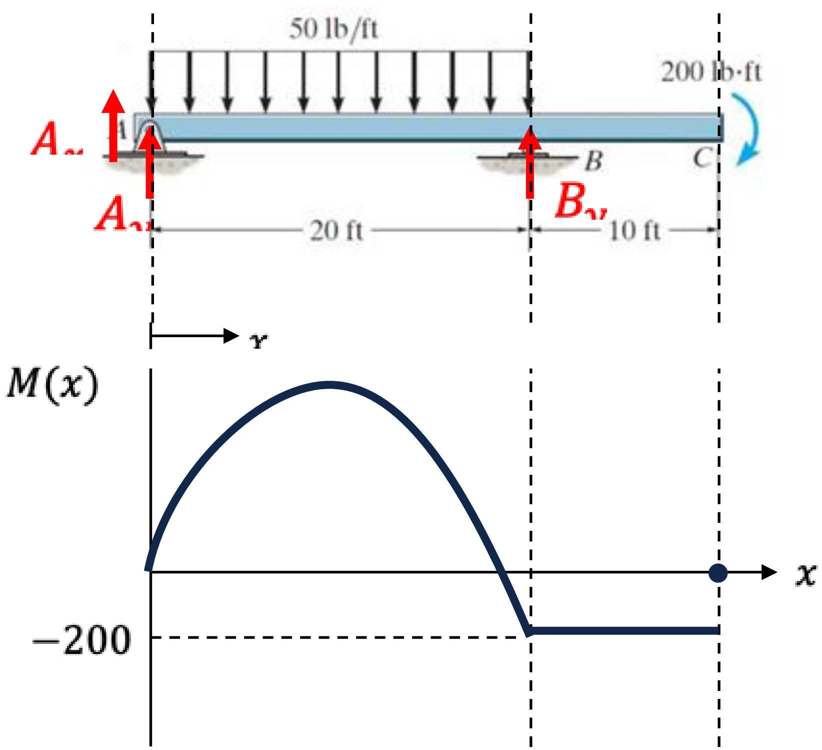

- region 1 ($0 \leq x < 20$ ft)

  $\therefore M(x) = -25x^2 + 490x$

- region 2 ($20 \leq x < 30$ ft)

  $\therefore M(x) = -200$

WEEK 10 – Internal Forces

- **Let's solve the first problem together!**
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  - Apply equations of equilibrium to solve for V and M as functions of x

c) Use the results from parts (a) and (b) to draw the shear and bending moment diagrams.